Transport phenomena in flows of granular materials

Abstract

Flowing granular materials are an example of a heterogeneous complex system away from equilibrium. As a result, their dynamics are still poorly understood. One canonical example is granular flow in a slowly-rotating container. Under some mild assumptions, the kinematics of the flow can be modeled and scalar mixing studied with the usual advection-diffusion paradigm. The shape of the container can induce chaotic dynamics, while the properties of the individual particles can lead to self-organization (demixing). The balance between these two effects leads to intricate persistent mixing patterns, which can be understood as "strange" eigenmodes. However, granular materials do not perform Brownian motion, so diffusion is observed in such systems because agitation (flow) causes inelastic collisions between particles. It has been suggested that axial diffusion of granular matter in a rotating drum might be "anomalous" in the sense that the mean squared displacement of particles follows a power law in time with exponent less than unity. Further numerical and experimental studies have been unable to definitively confirm or disprove this observation. We can show that such a "paradox" can be resolved using Barenblatt's theory of self-similar intermediate asymptotics. Specifically, we find an analytical expression for the instantaneous scaling exponent of a macroscopic concentration profile. Then, by incorporating concentration-dependent diffusivity in the model, we show the existence of a crossover from an anomalous scaling (consistent with experimental observations) to a normal diffusive scaling at very long times.